

Musical Mathematics

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ED 5601: Artistic Expression Explored Through Science and Mathematics

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Everyone listens and enjoys some kind of music, not everyone enjoys math. In order to create a more enthusiastic attitude towards functions and Algebra, I developed a unit connecting music with mathematics. The lessons described here are based on mathematical solutions to the composition of music. The students will be actively involved in graphing melodies, finding functions, transforming melodies and composing their own music. Students will develop their cooperative learning skills as they harmonize their compositions, add rhythm, analyze and graph their songs. The mathematical concepts covered will include; functions, transformations, fractions, patterns, graphing, patterns and probability.

Overview

Students use transformations of functions to compose music. They will represent a simple melody as a function, calculate several transformations, graph the transformations and play/listen to the transformed melodies. The students will be able to see and hear how the transformations changed the original function. The lesson will finish with students giving a performance of their own composition.

Materials:

- Instrument or virtual keyboard
- Tv/vcr
- Mozart and mathematics video
- Colored pencils
- Student handouts

Day 1

Start by opening a conversation about how musicians compose music. Describe how musicians and composers from Mozart to Michael Jackson have used mathematics both consciously and unconsciously. Play the video *Mozart and mathematics*, 50 minutes, which explores Mozart's mind, matter and mathematical genius.

Day 2

Discuss the video and how many ways mathematics can be connected to music. Take students to the computer lab and enter the site www.studyworksonline.com. There, students can read and follow the table of contents to learn about Mozart's Musical Dice Game. Students will have fun creating different melodies and might be impressed by the musical and mathematical genius of Mozart's game. It may be necessary to review concepts of probability before going into the computer lab.

Day 3-5

Begin by discussing the musical dice game and how they think Mozart was able to create the game and why the compositions sounded good no matter how they were arranged.

Put several bars on the overhead for the students to refer to. Many students can see that the variations are geometric transformations: rotations, reflections and translations.

Put the students into groups, be sure to put at least one musical person in each group.

Tell students that they will be composing using some of the same ideas that Mozart seemed to use. Refer students to the Number/Note Reference key and explain that musical notes are represented by dots on a staff but for the purpose of this activity we will represent them with numbers. Normally musicians identify notes by letters. Also, for this part of the lesson all notes will have equal length to them and we won't be concerned with rhythms.

Play or have a student play the original melody – the “meow meow song” from the commercial for Meow mix. A catchy tune everyone is sure to recognize! Give the students the numerical value for the sequence of notes and have the students record the notes on the staff for the original melody on Worksheet 1.

Sequence: 5 4 2 0 5 4 2 0 5 4 5 6 7 4 2 0



Play the tune again so students can see what they hear. They should notice that the higher the note sounds, the higher it will be placed on the staff. This melody will be $f(x)$, the original function. The numerical values for each note will be the function values of $f(x)$ and x will represent the order in which the notes are played: the first note $x = -8$, $f(x) = 5$, the second note $x = -7$, $f(x) = 4$ etc.. Discuss how this fits the definition of a function – input value x and output value $f(x)$. There is no “rule” or equation that relates the input and output and you may need to explain how it isn't necessary in the definition of a function. Have the students complete the Input/Output Table on Worksheet 1 for x and $f(x)$. (In order for later transformations to “work” the first note was assigned $x = -8$.) Next, have the students graph the original function in pencil on Worksheet 4 and 5 (yes they will graph it twice). Have the students connect the points to help them gain a visual image of the melody. You may need to explain this since the notes do not change gradually. Play the melody one more time while the students follow their graph.

Transformations – Part I

$$Y = f(x) + 2 \quad Y = 2 f(x) \quad Y = -f(x) \quad Y = f(x) - 3$$

Begin with the first transformation, ask students how they think these transformations might sound or look like. Have students calculate the transformations, record their results on the Input/Output Table of Worksheet 2 and fill in the bar for the new melody. Have students graph each transformation on Worksheet 4, using a different color pencil for each transformation. After each transformation, play the new melody several times as students follow their graphs and bars. After the students have completed all transformations have students record their observations and conclusions for each transformation on Worksheet 6. Play each transformation and discuss what each group has discovered. Students should write about the relationship between $f(x)$ and each transformation from both the graph and listening to the melody. You may also choose to include both geometric and musical vocabulary for each transformation (Garland, 1995, pp 69 – 78):

	<u>Geometric Transformation</u>	<u>Musical Transformation</u>
$Y = f(x) + 2$	vertical translation	transposition
$Y = 2 f(x)$	dilation (vertical)	
$Y = f(-x)$	reflection (across y-axis)	retrogression
$Y = f(x) - 3$	vertical translation	transposition

Transformations – Part II

$$Y = f(-x) \quad Y = (x + 1) \quad Y = f(2x) \quad Y = -f(-x)$$

This part of the activity explores transformations resulting from manipulating the input values. Help students insert function values since they may not see that for $x = -8$, $f(x + 1) = f(-8 + 1) = f(-7) = 2$. In some transformations you won't be able to obtain function values because the input function values because the input will be out of the specified domain of the function. This is a good opportunity to discuss the concepts of domain and range for each transformation. Again have students calculate each transformation, fill in the table, staff and graph. Play each melody several times after each transformation. Once all transformations have been completed students discuss the transformations and have students record their observations and conclusions for each transformation. Again, you may want to introduce geometric and musical vocabulary hear (Garland, 1995, pp 69 – 78):

	<u>Geometric Transformation</u>	<u>Musical Transformation</u>
$Y = -f(x)$	reflection (about the x-axis)	inversion
$Y = (x + 1)$	horizontal translation	repeat
$Y = f(2x)$	dilation (horizontal)	
$Y = -f(-x)$	rotation	retrograde inversion

Final Project

Students must use all the different kinds of transformations to make a piece of music. They need to represent the music graphically, musically, create an Input/Output table, and have it played in class by either a computer or students. Students can use Finale Note Pad 2002 to write the music and play it as well. They will also need to summarize how mathematics was used to create their music. Students should attempt to harmonize by combining transformations as well as add a rhythm component or lyrics.

Resources

Michigan Department of Education. (2000) *Michigan Curriculum Framework*.

Coda Music Technology (2001). *Finale Notepad 2003*. Available at:
www.codamusic.com

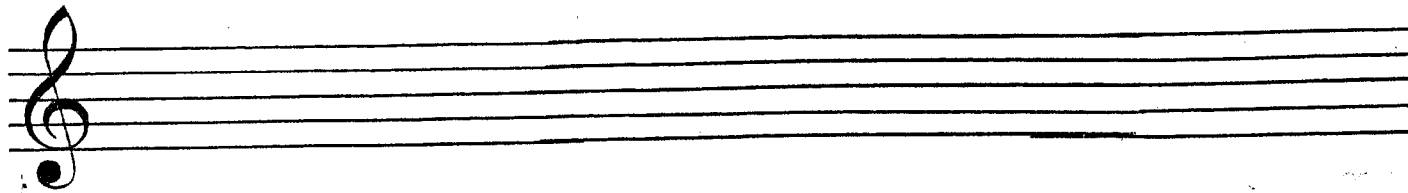
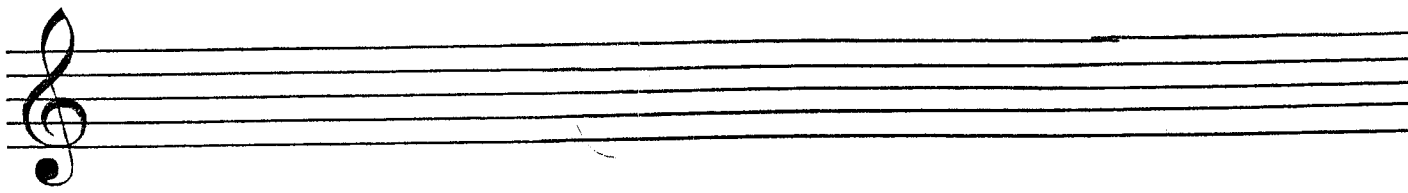
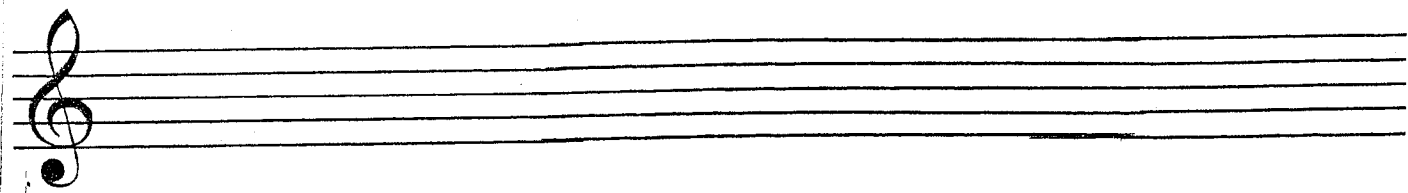
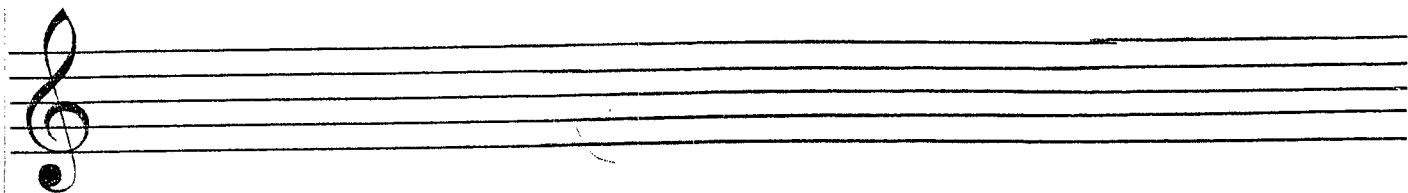
Garland, T.H., Kahn, C.H. (1995) *Math and Music*. Dale Seymour Publications.

Wayne County Regional Educational Service Agency. (2000). *Mozart, math and music*. Wayne, MI. Available at: REMC

Transformations – Part I

Input/Output Table

x	f(x)	f(x) + 2	f(x) - 3	2f(x)	-f(x)
-8					
-7					
-6					
-5					
-4					
-3					
-2					
-1					
0					
1					
2					
3					
4					
5					
6					
7					

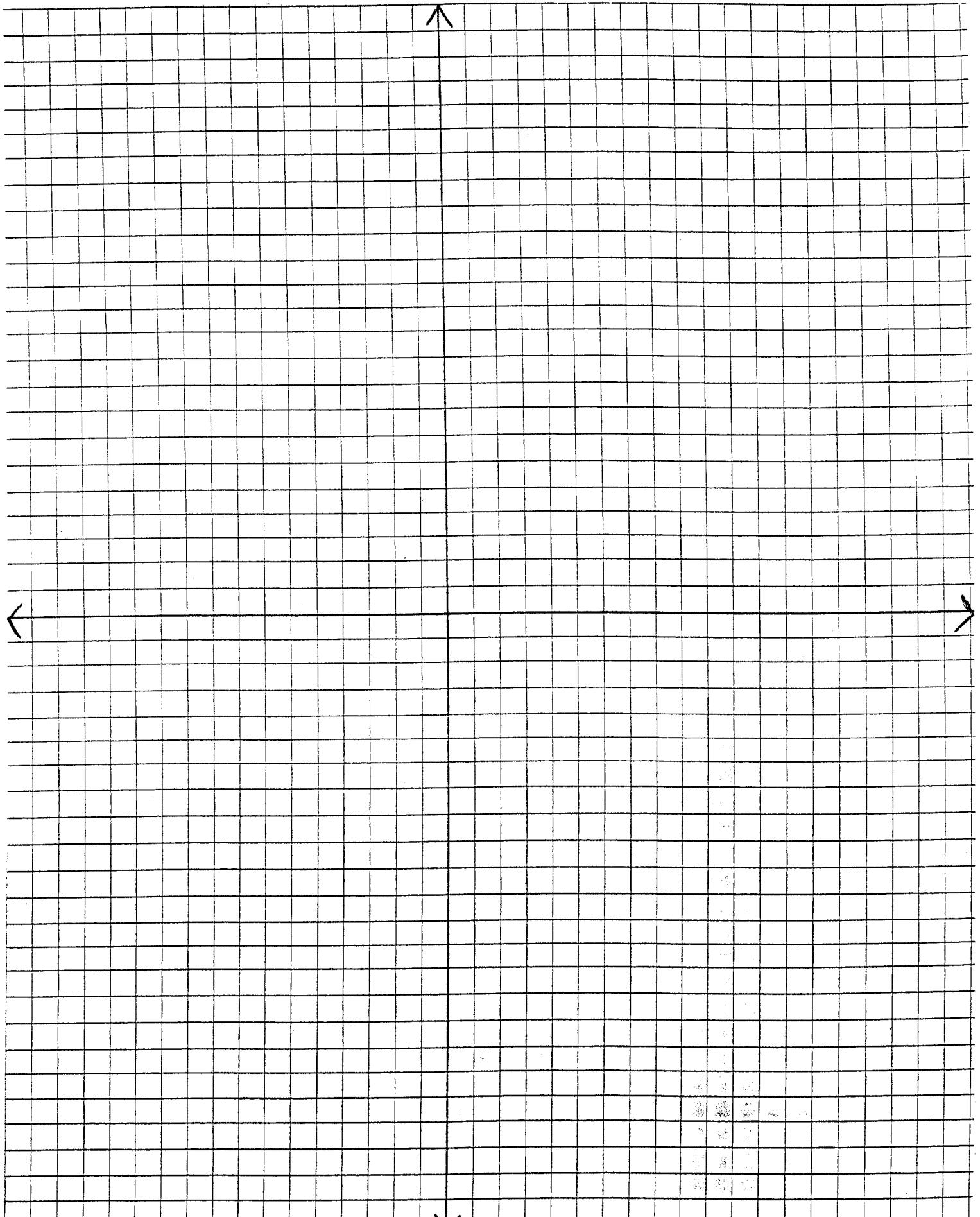


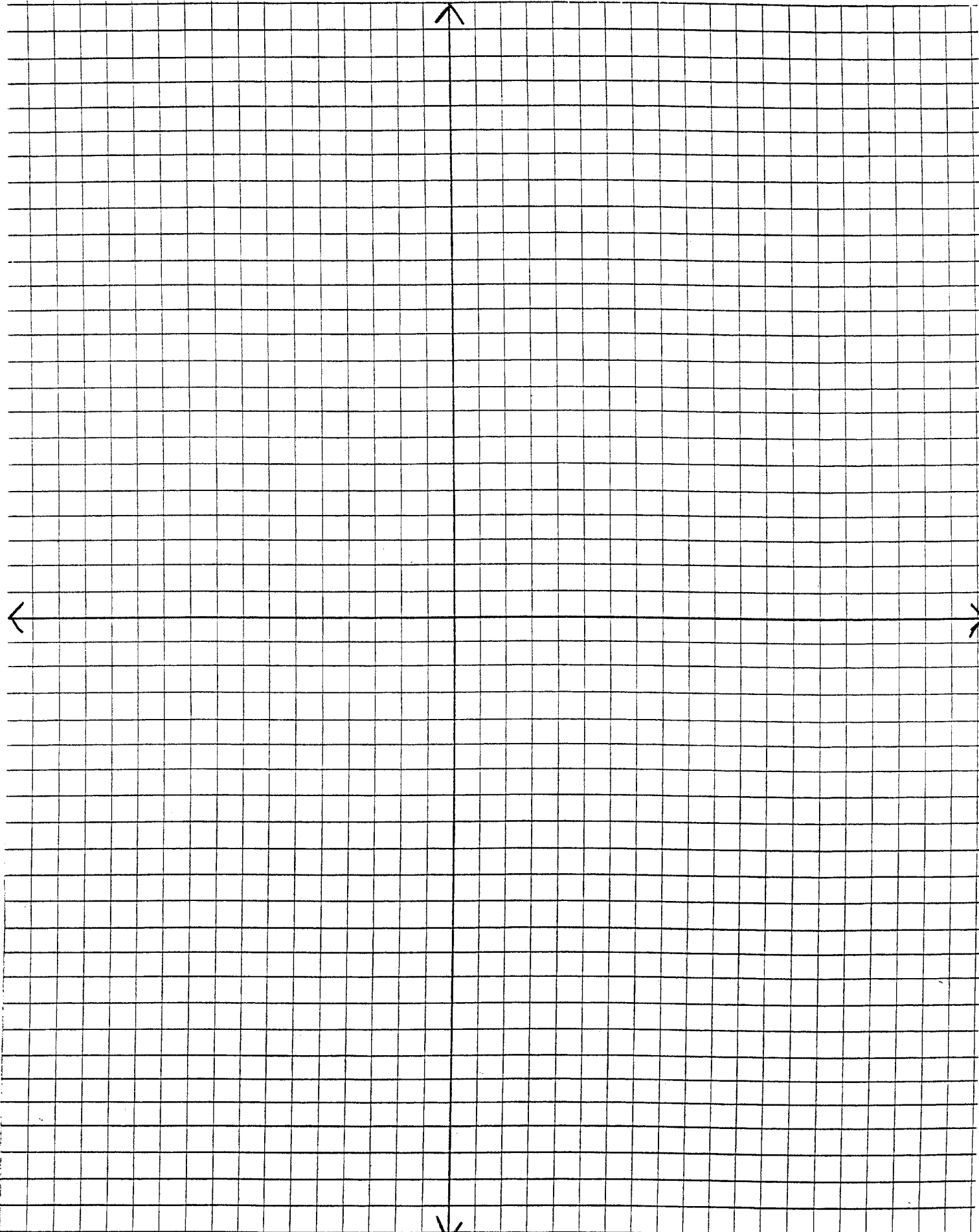
Transformations – Part II

Input/Output Table

x	f(x)	f(x + 1)	f(2x)	f(-x)	-f(-x)
-8					
-7					
-6					
-5					
-4					
-3					
-2					
-1					
0					
1					
2					
3					
4					
5					
6					
7					

Four blank musical staves, each with a treble clef, arranged vertically. The staves are empty, intended for writing musical notation.





Observations and Conclusions

<i>transformation</i>	<i>Observation</i>	<i>Conclusion</i>
$f(x) + 2$	Graph: _____ Melody: _____	
$f(x) - 3$	Graph: _____ Melody: _____	
$2f(x)$	Graph: _____ Melody: _____	
$-f(x)$	Graph: _____ Melody: _____	
$f(x + 1)$	Graph: _____ Melody: _____	
$f(2x)$	Graph: _____ Melody: _____	
$f(-x)$	Graph: _____ Melody: _____	
$-f(-x)$	Graph: _____ Melody: _____	

Transformations - Part I

"Meow meow song"

schuster

Piano

$f(x)+2$

The first system of music is for piano and is in 4/4 time. The treble clef staff contains a melody of quarter notes: G4, A4, B4, C5, B4, A4, G4, F4, E4, D4, C4. The bass clef staff contains whole rests. The label $f(x)+2$ is written in the left margin.

$f(x) - 3$

The second system of music is for piano and is in 4/4 time. The treble clef staff contains a melody of quarter notes: E4, F4, G4, A4, B4, C5, B4, A4, G4, F4, E4, D4, C4. The bass clef staff contains whole rests. The label $f(x) - 3$ is written in the left margin.

$2f(x)$

The third system of music is for piano and is in 4/4 time. The treble clef staff contains a melody of quarter notes: G4, A4, B4, C5, B4, A4, G4, F4, E4, D4, C4. The bass clef staff contains whole rests. The label $2f(x)$ is written in the left margin.

$-f(x)$

The fourth system of music is for piano and is in 4/4 time. The treble clef staff contains a melody of quarter notes: G4, A4, B4, C5, B4, A4, G4, F4, E4, D4, C4. The bass clef staff contains whole rests. The label $-f(x)$ is written in the left margin.

Transformations - Part II

schuster

Piano

$f(-x)$

This musical system is for the piano and is in 4/4 time. It consists of two staves: a treble clef staff and a bass clef staff. The treble staff contains a melodic line starting with a quarter rest, followed by quarter notes G4, A4, B4, C5, D5, E5, F5, G5, A5, B5, C6, D6, E6, F6, G6, A6, B6, C7, D7, E7, F7, G7, A7, B7, C8, D8, E8, F8, G8, A8, B8, C9, D9, E9, F9, G9, A9, B9, C10, D10, E10, F10, G10, A10, B10, C11, D11, E11, F11, G11, A11, B11, C12, D12, E12, F12, G12, A12, B12, C13, D13, E13, F13, G13, A13, B13, C14, D14, E14, F14, G14, A14, B14, C15, D15, E15, F15, G15, A15, B15, C16, D16, E16, F16, G16, A16, B16, C17, D17, E17, F17, G17, A17, B17, C18, D18, E18, F18, G18, A18, B18, C19, D19, E19, F19, G19, A19, B19, C20, D20, E20, F20, G20, A20, B20, C21, D21, E21, F21, G21, A21, B21, C22, D22, E22, F22, G22, A22, B22, C23, D23, E23, F23, G23, A23, B23, C24, D24, E24, F24, G24, A24, B24, C25, D25, E25, F25, G25, A25, B25, C26, D26, E26, F26, G26, A26, B26, C27, D27, E27, F27, G27, A27, B27, C28, D28, E28, F28, G28, A28, B28, C29, D29, E29, F29, G29, A29, B29, C30, D30, E30, F30, G30, A30, B30, C31, D31, E31, F31, G31, A31, B31, C32, D32, E32, F32, G32, A32, B32, C33, D33, E33, F33, G33, A33, B33, C34, D34, E34, F34, G34, A34, B34, C35, D35, E35, F35, G35, A35, B35, C36, D36, E36, F36, G36, A36, B36, C37, D37, E37, F37, G37, A37, B37, C38, D38, E38, F38, G38, A38, B38, C39, D39, E39, F39, G39, A39, B39, C40, D40, E40, F40, G40, A40, B40, C41, D41, E41, F41, G41, A41, B41, C42, D42, E42, F42, G42, A42, B42, C43, D43, E43, F43, G43, A43, B43, C44, D44, E44, F44, G44, A44, B44, C45, D45, E45, F45, G45, A45, B45, C46, D46, E46, F46, G46, A46, B46, C47, D47, E47, F47, G47, A47, B47, C48, D48, E48, F48, G48, A48, B48, C49, D49, E49, F49, G49, A49, B49, C50, D50, E50, F50, G50, A50, B50, C51, D51, E51, F51, G51, A51, B51, C52, D52, E52, F52, G52, A52, B52, C53, D53, E53, F53, G53, A53, B53, C54, D54, E54, F54, G54, A54, B54, C55, D55, E55, F55, G55, A55, B55, C56, D56, E56, F56, G56, A56, B56, C57, D57, E57, F57, G57, A57, B57, C58, D58, E58, F58, G58, A58, B58, C59, D59, E59, F59, G59, A59, B59, 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